A Digital Elevation Model for Egypt by Collocation

***Dr. Maher Mohamed Amin*[1]** ***Dr. Saadia Mahmoud El-Fatairy*[1]**
***Eng. Raaed Mohamed Hassouna*[2]**
[1] Lecturer of Surveying, Surveying Department, Shoubra Faculty of Engineering, Zagazig University
[2] Assistant Lecturer, Civil Engineering Department, Faculty of Engineering in Shebin El-Kom, Menoufia University

Abstract:
In the current investigation, a 5'x5' digital terrain model for Egypt was computed by collocation, based on the available local height data and the high-resolution harmonic model GTM3a. Exploiting the dominant moderately varying Egyptian territory, the global high frequency topographic harmonic model was removed prior to the prediction into a grid and a posteriori restored at the grid nodes. A removal and restoration of a suitable trend surface, based on the residual height data, has also contributed to the smoothing strategy. According to the limited coverage and resolution of the available height data, the global-wise accuracy of the resulting DEM was found to be ± 29 meters.

1 Introduction

It is well known that the Egyptian territory is of a dominant moderate topographic variation. Exceptions are the west and east-southern boundaries, the Red Sea Mountains series and the Sinai Mountains. This fact implies that the terrain in Egypt forms in general a surface of a relatively moderate smooth nature. Hence, such a surface could be handled in the same way as the (much more smoother) anomalous gravitational quantities such as the geoidal heights. Also, the physical nature of the height above mean sea level, characterized by the relevant geopotential number and the mean gravity along the local plumb line, this nature makes sense of manipulating the (orthometric) height, at least mathematically, as if it were of a residual smooth nature.

As the aim of this study was to establish a digital terrain model for Egypt by collocation, it was intended to use the high-resolution global harmonic model GTM3a as representing the low-resolution (long wavelength) part of the height signal. The effect of this harmonic model, along with a suitable trend surface, was removed from the local MSL height data prior to the LSC interpolation into a grid. The data points are either vertical control benchmarks, GPS-geoid stations or gravity stations with known leveled heights relative to the local Egyptian vertical datum.

The scattered “smoothed” residual height data were then used as input for the least-squares prediction into the nodes of a 5'x5' grid. The GTM3a height signal as well as the trend surface effect were then added back (restored) to the gridded residual height values in order to end up with the full 5'x5' digital elevation model for the region bounded by ($21°N \le \varphi \le 33°N$; $24°E \le \lambda \le 37°E$), covering Egypt and exceeding its boundaries.

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The computed terrain model was evaluated at two sets of points. The first group was the height data points used in the prediction of the model in order to express the internal accuracy. The second group was not used as data so as to give a statement about the achieved external accuracy of the estimated terrain model. The evaluation showed a reasonable internal and external accuracy of the target model, relative to the limited resolution and coverage of the input data.

2 Background

As a gravimetric quantity, the geoid undulation can be expanded in terms of global spherical harmonic coefficients as follows

$$N(\psi,\lambda,r) = (GM/r\gamma) \sum_{n=0}^{nmax} (a/r)^n \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\psi) \qquad (1)$$

where

ψ the geocentric latitude,
λ the geodetic longitude,
r the geocentric radius to the geoid,
$\gamma(\psi,r)$ the normal gravity implied by the reference ellipsoid,
GM the Earth mass gravitational constant,
a the equatorial radius,

$\bar{C}^*_{nm}, \bar{S}_{nm}$ the fully normalized spherical harmonic coefficients of degree n and order m, reduced for the even zonal harmonics of the reference ellipsoid,

$\bar{P}_{nm}(\sin\psi)$ the fully normalized associated Legendre function of degree n and order m,
n_{max} the maximum degree of the geopotential harmonic model.

In spherical approximation, both the geoid and the reference ellipsoid are geometrically represented by the mean terrestrial sphere of radius R = 6371 km. So, the equatorial radius a is replaced by the mean radius of the Earth R, the point normal gravity γ by the mean Earth gravity GM/R^2, the rigorous geocentric radius r (at the geoid) is replaced by R and the geocentric latitude ψ by the geodetic latitude φ (Heiskanen and Moritz, 1967). Hence, substituting these values in Eq. (1), the geoidal height can be expanded in the following surface harmonics

$$N(\varphi,\lambda) = R \sum_{n=0}^{nmax} \sum_{m=0}^{n} (\bar{C}^*_{nm} \cos m\lambda + \bar{S}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\varphi) \qquad (2)$$

The height of the Earth topography can also, in analogy to the geoid in Eq. (2), be expanded in surface harmonic series (Burša, 1971 and Tscherning and Forsberg, 1986). This is based on the assumption that the physical surface of the Earth is gentle enough so that it can be treated as if it were a gravimetric quantity. Namely, the height H of the topography above mean sea level can be expressed as

$$H(\varphi,\lambda) = R \sum_{n=0}^{nmax} \sum_{m=0}^{n} (\bar{A}_{nm} \cos m\lambda + \bar{B}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\varphi) \tag{3a}$$

where $\bar{A}_{nm}$ and $\bar{B}_{nm}$ are the fully normalized (unitless) harmonic coefficients (of degree n and order m) of the Earth topographic height above mean sea level, $P_{nm}(\sin\varphi)$ is the usual fully normalized associated Legendre function and n_{max} is the maximum degree of the global topographic harmonic model. Eq. (3a) can be rearranged as follows

$$H(\varphi,\lambda) = \sum_{n=0}^{nmax} \sum_{m=0}^{n} (\bar{D}_{nm} \cos m\lambda + \bar{E}_{nm} \sin m\lambda)\, \bar{P}_{nm}(\sin\varphi) \tag{3b}$$

with

$$\bar{D}_{nm} = R.\, \bar{A}_{nm}$$

$$\bar{E}_{nm} = R.\, \bar{B}_{nm}$$

So, both $\bar{D}_{nm}$ and $\bar{E}_{nm}$ are still fully normalized harmonic coefficients but have now meter dimension. This reformulation is intuitively useful, since the topographic height harmonic coefficients are computed in practice directly in meter units as will be clarified below.

Theoretically, if the Earth terrain height (or bathymetry in marine areas) $H(\varphi,\lambda)$ is continuously available over the whole globe, then the corresponding fully normalized harmonic coefficients can be determined by (Burša, 1971)

$$\bar{D}_{nm} = (1/4\pi R^2) \iint_e H(\varphi,\lambda) \,.\, \cos m\lambda\, \bar{P}_{nm}(\sin\varphi).R^2 \cos\varphi\, d\varphi\, d\lambda$$

$$\bar{E}_{nm} = (1/4\pi R^2) \iint_e H(\varphi,\lambda) \,.\, \sin m\lambda\, \bar{P}_{nm}(\sin\varphi).\, R^2 \cos\varphi\, d\varphi\, d\lambda \tag{4}$$

Practically, however, the formal global terrain height data is often global mean terrain heights of a specific resolution, say I x 2I global equiangular grid cells. Then, the integrals in Eq. (4) will be replaced with discrete summations as follows (Fan, 1998)

$$\bar{D}_{nm} = (1/4\pi R^2) \sum_{i=1}^{I} \sum_{j=1}^{2I} H(\varphi_i,\lambda_j) \,.\, \cos m\lambda_j\, \bar{P}_{nm}(\sin\varphi_i).R^2 \cos\varphi_i\, \Delta\varphi\, \Delta\lambda$$

$$\bar{E}_{nm} = (1/4\pi R^2) \sum_{i=1}^{I} \sum_{j=1}^{2I} H(\varphi_i,\lambda_j) \cdot \sin m\lambda_j \, \bar{P}_{nm}(\sin\varphi_i). \, R^2 \cos\varphi_i \, \Delta\varphi \, \Delta\lambda \qquad (5)$$

where φ_i, λ_j are the geodetic coordinates of the running block center and $\Delta\varphi$, $\Delta\lambda$ are the (equal) latitude and longitude grid intervals.

GTM3a is a global high-resolution harmonic model of the Earth topography. It was computed from the global 5'x5' ETOPO5 mean elevation data and. The fully normalized coefficients D_{nm} and E_{nm} were computed in meter as given by Eq. (5). The coefficients were evaluated up to degree and order 1800. Following the $180^o/\psi^o$ theoretical rule of thumb, the recoverable resolution of such a mean elevation grid should be up to degree and order 2160. However, the numerical instability of evaluating the fully normalized Legendre function for degrees more than 1800 hampered the evaluation of coefficients more than 1800 degree (Wenzel, 1998).

3 Digital terrain modeling considerations

Least-squares collocation is an efficient interpolation technique, which provides minimum standard errors for the predicted signals based on the observational data and its noise (Moritz, 1980). So, it was intended to utilize this technique to digitally model the terrain in Egypt. An essence is the height covariance function that describes the variation and covariance of the height signal at pairs of stations as function of the distance between them. Due to the rough nature even of any gently varying terrain relative to any gravimetric quantity, the maximum smoothing should be aimed at, via the removal of certain (known effects) and finally restoring such effects to the residual predicted height signal. This is essential in order to obtain the best quality of the resulting elevation model. There are many smoothing methods commonly used in practice, such as, the removal of the mean value, the removal of the long wavelength content via suppressing a suitable harmonic model of the topographic height and the removal of a suitable trend for the residual heights.

In order to obtain a smooth version of the used height data, the low frequency height contribution from the GTM3a model, up to degree and order 1800, was firstly removed from the data. Particularly, the harmonic model height values were computed point-wise at the discrete height data points (Eq. (3b)) and were subtracted from the heights of these points. For a further smoothing of the residual height data, a polynomial trend surface was also removed from the residual data. This trend could also account for a long wavelength error due to the removal of the harmonic model. In fact, several polynomial degrees were investigated regarding the smoothness of the final residual data. It was found that a further removal of a 3rd degree polynomial gave the minimum residual height variance. According to this trend surface, the height trend part for any point having (arbitrary local) curvilinear horizontal coordinates, e & n, is given as

$$H(e,n) = a_0 + a_1 e + a_2 n + a_3 en + a_4 e^2 + a_5 n^2 + a_6 e^2 n + a_7 n^2 e + a_8 e^3 + a_9 n^3, \qquad (6a)$$

with e and n taken as

$$e = (\lambda - \lambda_{min}) \cos\varphi \tag{6b}$$
$$n = \varphi - \varphi_{min} \tag{6c}$$

where φ, λ are the geodetic coordinates of the point of interest and φ_{min}, λ_{min} are the geodetic coordinates of the west-southern most data point. The ten coefficients are estimated via a least-squares fitting of the surface to the residual data.

Table (1) shows the statistics of the original height data, the statistics after removal of the mean value, after being reduced by the GTM3a harmonic model, after the removal of the mean of the GTM3a smoothed data and finally the statistics of the data after the removal of both GTM3a and the third degree trend surface. From this table, it is clear that in general the removal of the mean has a conventional smoothing effect. Moreover, the GTM3a harmonic model has a great amount of low frequency information regarding the Egyptian territory, and hence has a great smoothing effect (about 78% decrease in absolute mean, 32% in Std. Dev. and 52% in RMS). It is also clear that the 3^{rd} degree trend surface further removes considerable low degree information and a noticeable smoothing action (final Std. Dev. decrease is about 39% and about 59% in terms of RMS). The table, of course, ensures the fact that the height signal, even if it has moderate gentle variation, is much rougher than any gravimetric signal.

Table (1): Statistics of the original and reduced height data (units: meter)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Height	172.525	153.882	231.157	-38.800	1526.929
Height - mean	0.000	153.882	153.846	-211.325	1354.404
Height – GTM3a	-38.298	103.951	110.758	-578.649	689.493
(Height – GTM3a) - mean	0.000	103.951	103.926	-540.351	727.791
Height–GTM3a–3^{rd} degree trend	0.000	93.285	93.263	-620.717	705.864

Figure (1) plots the corresponding empirical isotropic covariance functions. To estimate an isotropic covariance function empirically at a spherical distance, ψ, the product sum average of pairs of (residual) height values, relevant to point pairs having spacing $\psi-\Delta\psi/2\leq\psi'\leq\psi+\Delta\psi/2$, was evaluated. Both $\Delta\psi$ and the ψ increment were chosen to be 2 minutes of arc and 100 covariance values (at 100 ψ values) were evaluated. Of course, such a function is dependent only on the spherical distances between pairs of stations, implying the invariance under a rotation of the data points group. An an-isotropic covariance function would be dependent on the positions of stations (Tscherning, 1999).

The trends of the covariance functions ensure the same results shown in Table (1). The covariance function of the height data, smoothed via mean value removal, is very smooth compared with that of the original data. Figure (1) shows also the greater smoothing provided by the GTM3a removal and the considerable additional smoothness implied by the removal of the 3^{rd} degree trend surface. The smoothness is, of course, expressed by the great decrease in the signal variance and correlation length, compared to those pertaining to the original data covariance function.

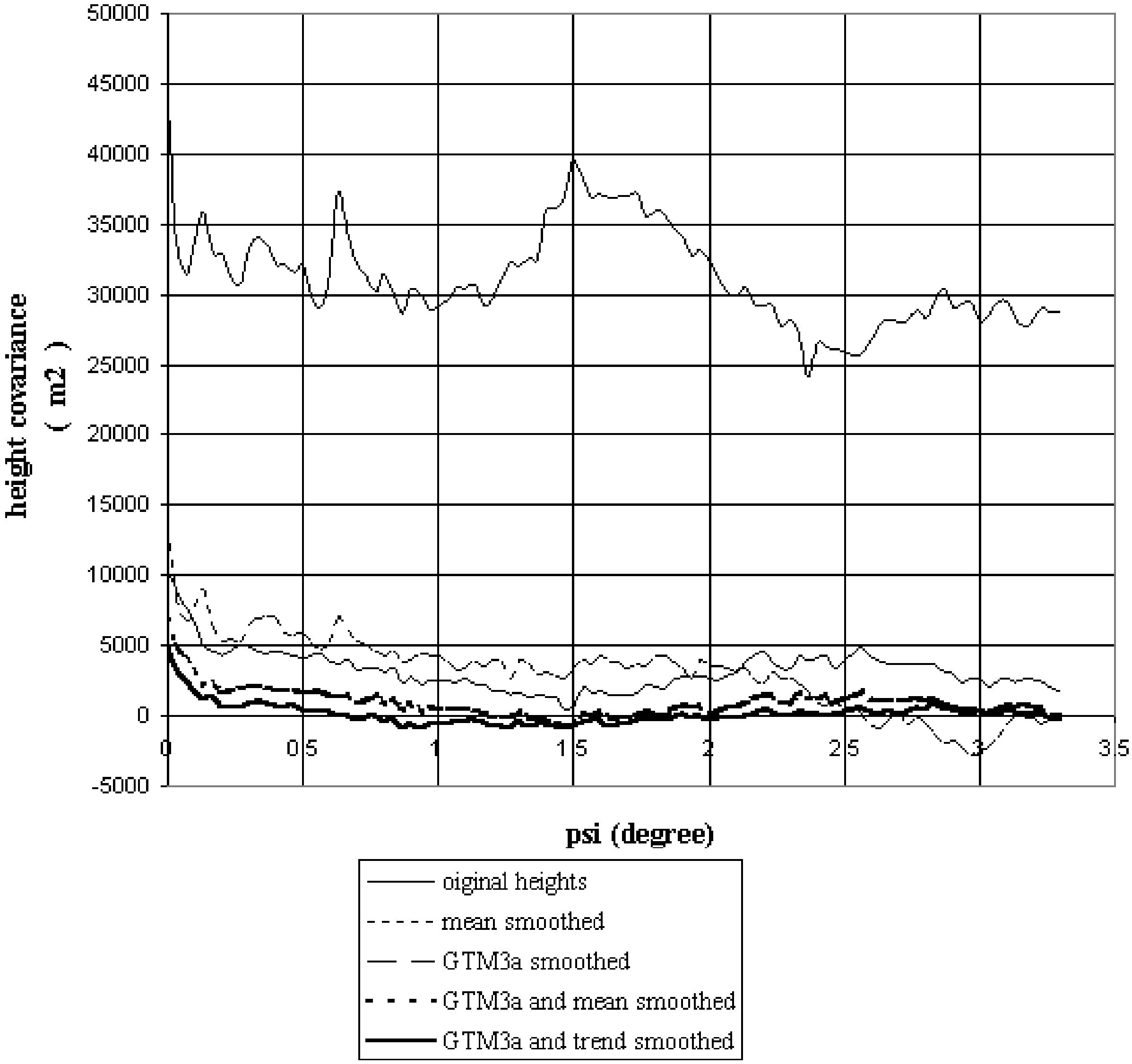

Figure (1): Comparison among the original and residual height data empirical covariance functions

Based on the above, it was decided to use as input for collocation the data smoothed by both the GTM3a model and the 3rd degree trend. Consequently, it was necessary to model the corresponding empirical covariance function. The residual height variance C_0 and the correlation length ξ are used to define a suitable analytical covariance function that was utilized during the LSC solution. The second order Markov model was used, in which the covariance in terms of the mutual spherical distance, ψ, between pairs of stations is given as

$$cov(\psi) = C_0(1 + \psi/\alpha) . e^{(-\psi/\alpha)}, \text{ with } \alpha = 0.595\xi \quad (7)$$

where the above value of α results from the condition that the covariance equals $C_0/2$ at distance $\psi = \xi$. It is well known that different covariance models sharing the same essential parameters give essentially the same results for prediction (Abd-Elmotaal and El-Tokhey, 1997). Finally, using the covariance model (Eq. (7)), the residual data and the data noise, the LSC solution resulted in the residual 5'x5' DEM grid. The predicted signals and their associated error estimates are given by the well known expressions

$$S = C_{sh}.(C_{hh} + N_{hh})^{-1} \,.\, h, \qquad (8a)$$

$$N_{ss} = C_{ss} - C_{sh}.(C_{hh} + N_{hh})^{-1} \,.\, C_{sh}^{T}, \qquad (8b)$$

with

S	the vector of estimated (residual height) signals,
C_{sh}	the cross-covariance matrix between the signals S and the (residual height) observations h,
C_{hh}	the covariance matrix of the (residual height) observations,
N_{hh}	the error variance-covariance matrix of the (residual height) observations,
h	the vector of (residual height) observations,
N_{ss}	the estimated error variance-covariance matrix of the estimated signals S,
C_{ss}	the covariance matrix of the signals S.

Due to the relatively rough nature of the height signal, it was intended to use a data point circular window around each prediction point. This procedure was followed, in order to account for the immediate topographic variations in the vicinity of each prediction point. Namely, at each prediction point, only the nearest N_{max} data points are used for the prediction. A selected value of 25 for N_{max} was found to be suitable. The harmonic model GTM3a and the trend height components were then added back (restored) to the residual grid values. These components were evaluated , using Eq. (3b) and Eq. (6), respectively, in terms of the geodetic coordinates of the grid nodes. The 5'x5' DEM grid covers the region bounded by ($21°N \leq \varphi \leq 33°N$; $24°E \leq \lambda \leq 37°E$).

4 Results

The ETOPO5 data, used originally for the solution for the harmonic coefficients of the topography, contained bathymetry data in marine regions. Consequently, the GTM3a harmonic model resulted in bathymetric depths at such regions. Figure (2a) is a contour map for the GTM3a 5'x5' topographic height grid. From this map, it is clear how this harmonic model possesses reasonable long wavelength topographic and bathymetric features for Egypt. In Figure (2b), the same information is plotted with the exception that the bathymetry in sea was conventionally assigned zero height. Figure (2b) shows, using terrain contours only, an expected map for Egypt, regarding the Red and Mediterranean Sea coasts.

Figure (3) plots a contour map for the trend part that was restored back to the residual grid heights obtained by LSC. Clearly, significant long wavelength information has been accounted for via this 3rd degree polynomial trend in the regions that had no data. Figure (4) shows a contour map of the sum of the residual grid values and the trend part values, that is the full component DEM minus the GTM3a harmonic model component. Comparing Figure (4) with Figure (3), the height data short wavelength contribution is very clear in Figure (4). This short wavelength information is more pronounced in mountainous areas, where the height signal changes (spatially) rapidly.
However, the limited number of height data points does not provide an ideal high-resolution data sample. Table (2) shows the statistics of the 5'x5' grid GTM3a heights, combined (residual + trend part) heights and the final height values.

Table (2): Statistics of the DEM elements (units: meter)

Item	Mean	Std. Dev.	RMS	Min.	Max.
GTM3a topography height	88.808	758.729	763.892	-3147.715	1843.014
Final DEM-GTM3a heights	46.747	122.716	131.316	-412.798	609.148
Final DEM height value	135.556	746.798	758.985	-3002.926	2152.436

The final contour map for the final 5'x5' digital terrain model for Egypt is plotted in Figure (5a). Comparing this figure with Figure (2a), one could recognize how well the GTM3a harmonic model along with the trend surface support the main features of the DEM. In addition, the additional short wavelength features implied by the observed height data set are easy to notice. Figure (5b) represents the same contour map, but neglecting the sea bathymetry as in Figure (2b). Finally, Figure (6) is a 3D-surface plot of Figure (5b).

The digital terrain modeling accuracy was evaluated firstly at the same data points used as observations in the LSC solution. These data points were selected, out of the total available height data points, to be the nearest points to the nodes of a 2'x2' grid covering the region of interest. The statistics of the discrepancy between the original observations and the height predictions at the same points are shown in Table (3). The terrain modeling accuracy was also checked at height data points that were not used as input for the LSC solution. The statistics between the observed and predicted height values at these points are also shown in Table (3). Obviously, the relatively rough terrain features in some regions and the lack of high-resolution height data points with uniform coverage allover Egypt, are responsible for this 29m standard deviation of differences. No detection of outliers procedure was performed, since it is very difficult to distinguish whether a great discrepancy is due to the existence of gross errors, not properly accounted for height signal roughness or a shortcoming in data resolution and coverage (Tscherning, 1982). Finally, in light of the lack of a (quasi) perfect data sample that best represents the target DEM resolution, the obtained results could be regarded as satisfactory.

After being used for assessing the external accuracy of the developed DEM, the check points were added to the previous data points, in order to develop a newer model covering the same geographic region and with the same resolution. Thus, this LSC solution, which also utilized the remove-restore technique exactly as described above, exploited the total available height data set. The final topographic map, relevant to that solution, is plotted in Figure (7), again neglecting sea bathymetry. The resulting model accuracy was also evaluated at the (used) new data points. Table (4) shows the statistics of the differences between the observed and predicted values at these points. Of course, a great improvement (about 77%) has been achieved in terms of the standard deviation and the RMS over the relevant values in Table (3), due to the use of those points as additional data. In addition, a great decrease in the mean, minimum and maximum difference can be noticed. This fact supports the intuition, that a huge amount of well-distributed new height data points, would certainly yield an excellent accuracy of the computed terrain model.

Table (3): Statistics of the differences between the observed and predicted height values at the same input data points and at independent check points (units: meter)

Item	Type	Mean	Std. Dev.	RMS	Min.	Max.
Observed heights-predicted heights	Input data points	0.000	0.193	0.193	-3.187	2.771
	Check points	1.139	29.017	29.009	-199.984	303.854

Table (4): Statistics of the differences between observed and predicted height values at the added new data points (units: meter)

Item	Mean	Std. Dev.	RMS	Min.	Max.
Observed heights-predicted heights	-0.346	6.713	6.714	-93.506	60.286

5 Concluding remarks

The digital elevation model developed for Egypt in the current study has a reasonable accuracy in light of the available data resolution and coverage. The GTM3a topographic harmonic model proved to provide excellent low degree information for the Egyptian territory. Also, the removal of the 3rd degree polynomial contributed effectively to the high pass filtering of long wavelength information. The height data, in turn, succeeded more or less in recovering the high frequency height signal, based on the limited given data number, resolution and coverage. It is recommended to use the developed terrain model in modeling the terrain effects for detailed gravimetric geoid determination. Also, the LSC height interpolation procedure, via the remove-restore technique, could be more effective when using a huge number and a better coverage of the height data points. This model can be used to produce a national topographic map of Egypt with a scale of 1 : 100 000 .

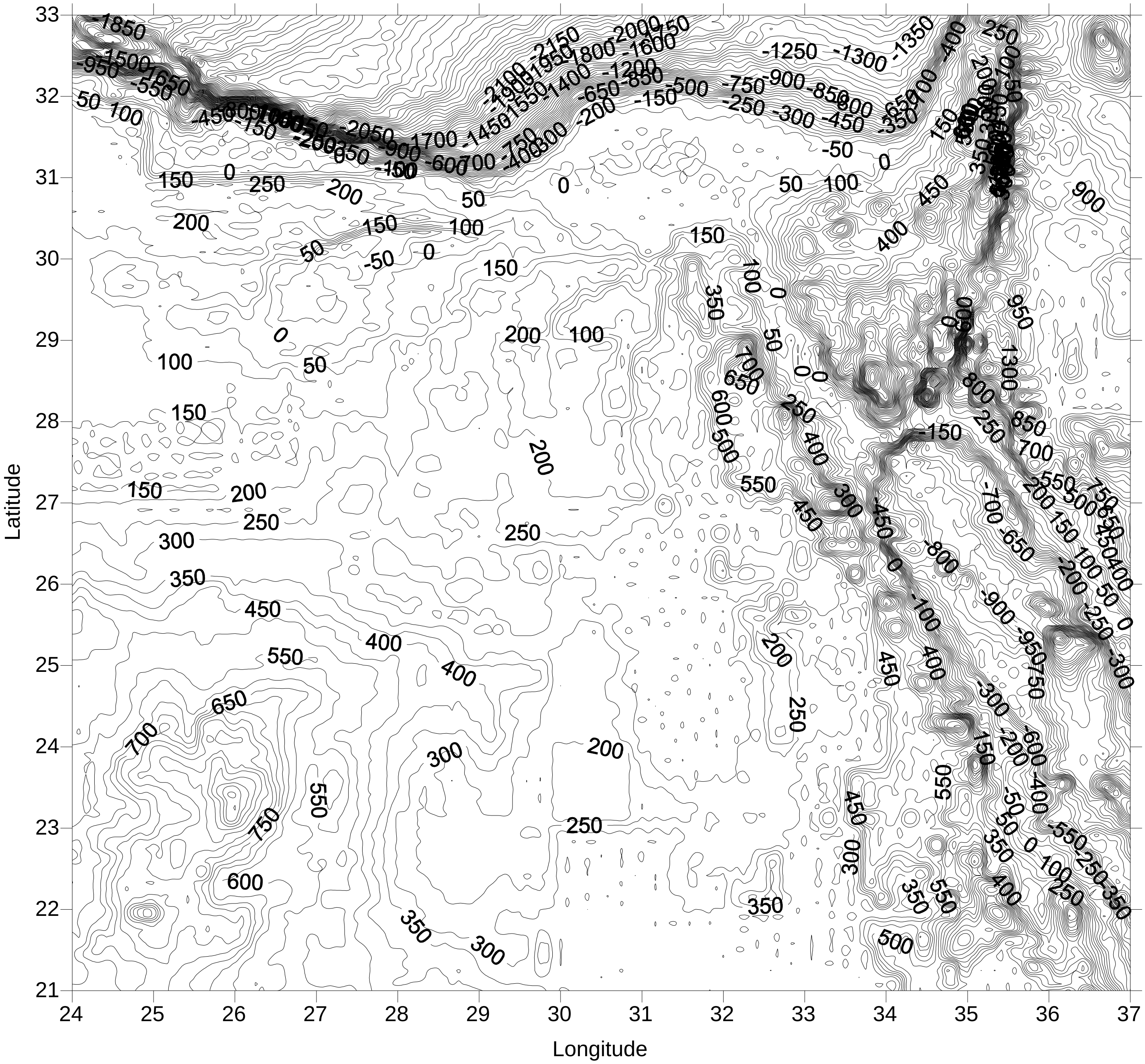

Figure (2a): GTM3a topographic contour map (interval: 50m)

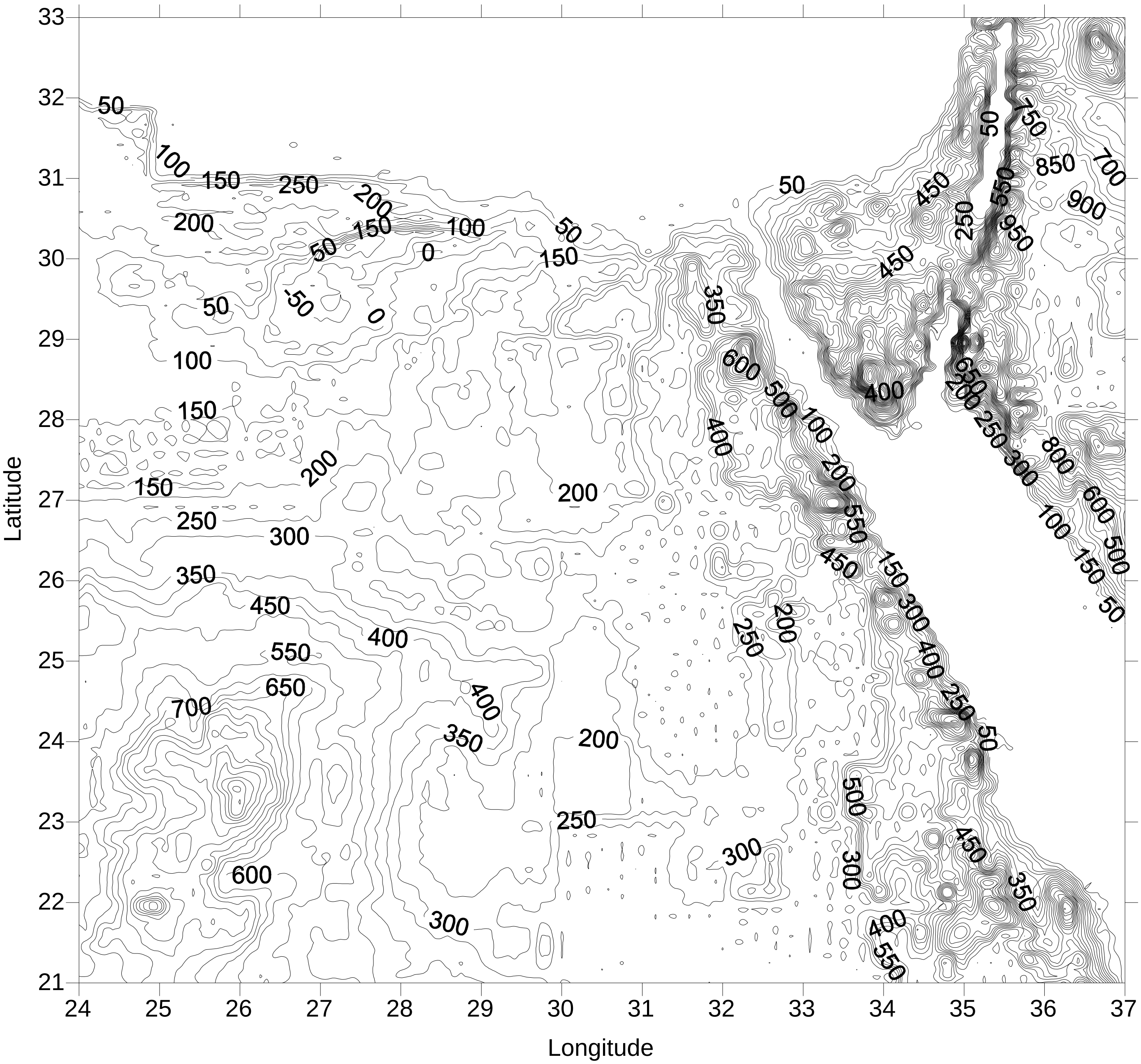

Figure (2b): GTM3a topographic contour map
(sea bathymetry neglected) (interval: 50m)

Figure (2)

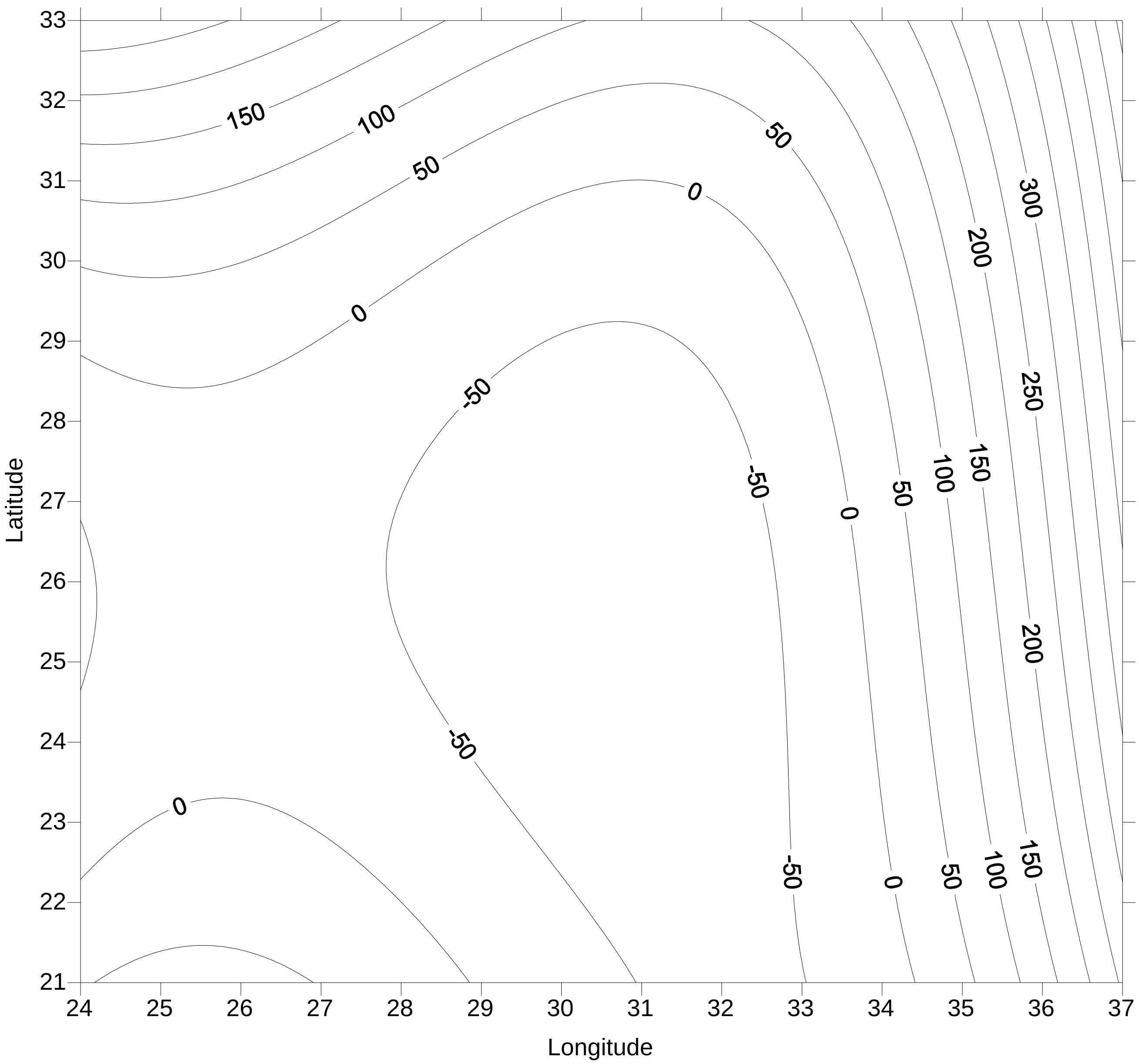

Figure (3): Trend part grid contour map (interval: 50m)

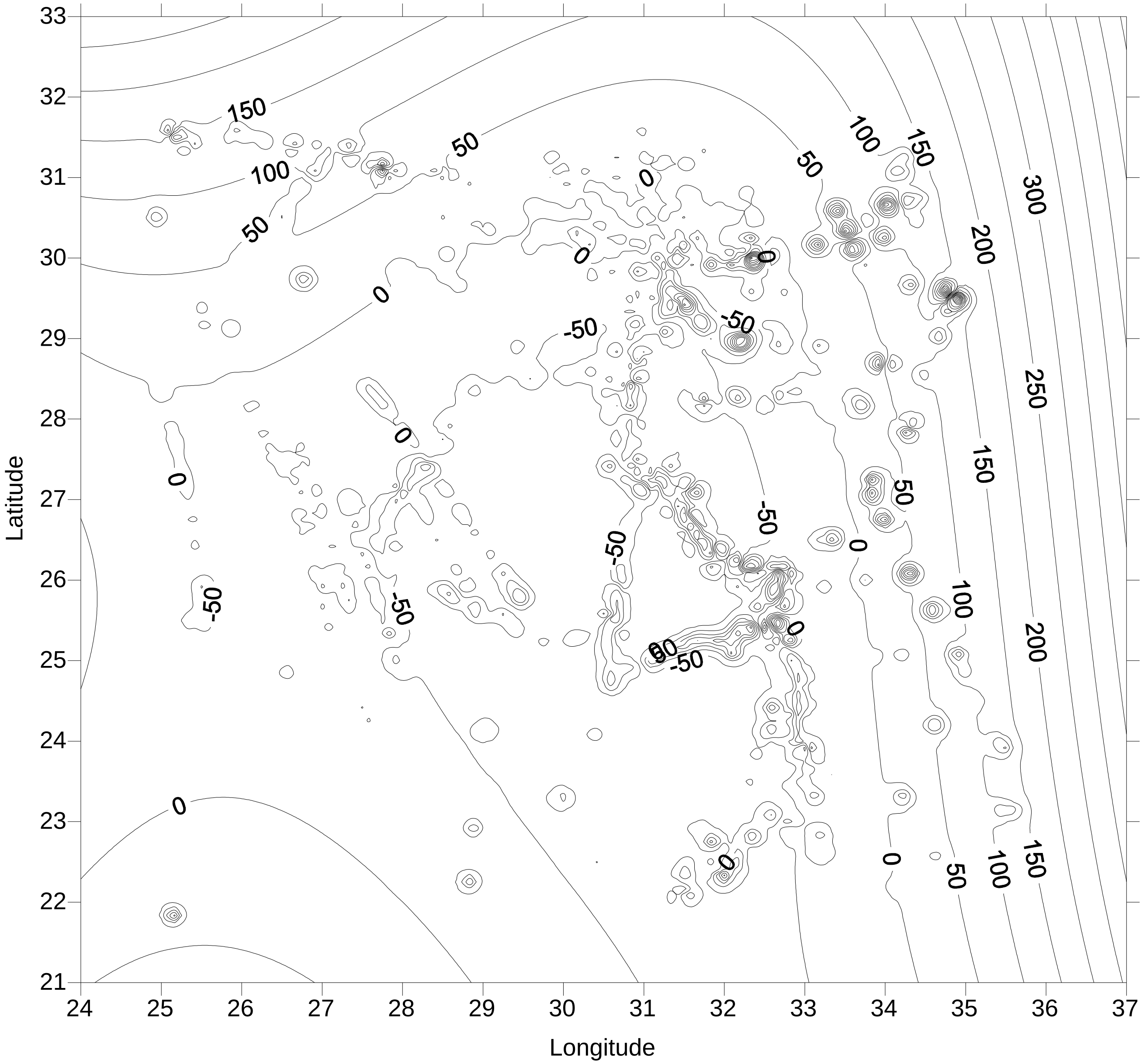

Figure (4): (Residual + trend part) height grid contour map (interval: 50m)

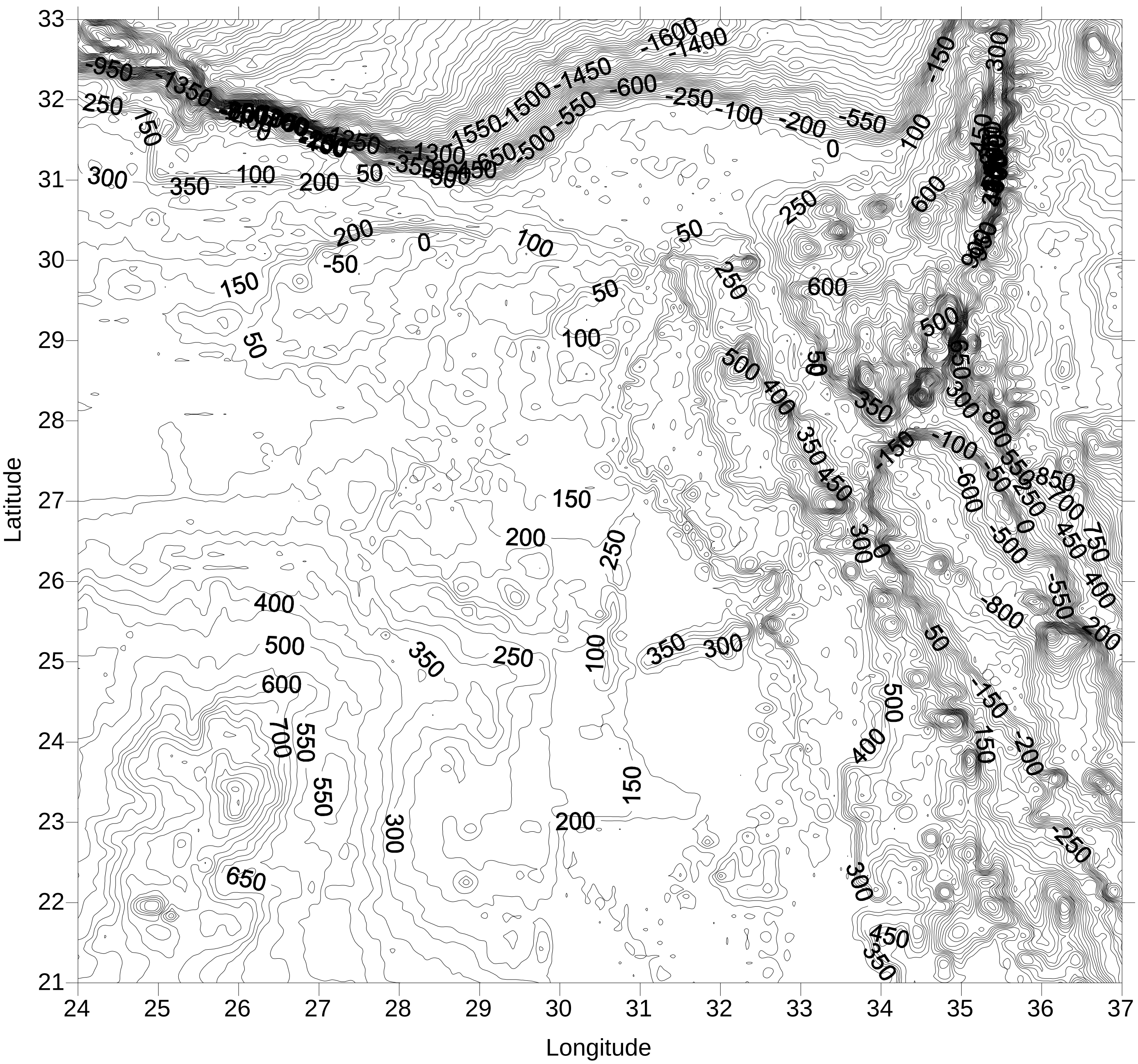

Figure (5a): Final topographic contour map for Egypt (interval: 50m)

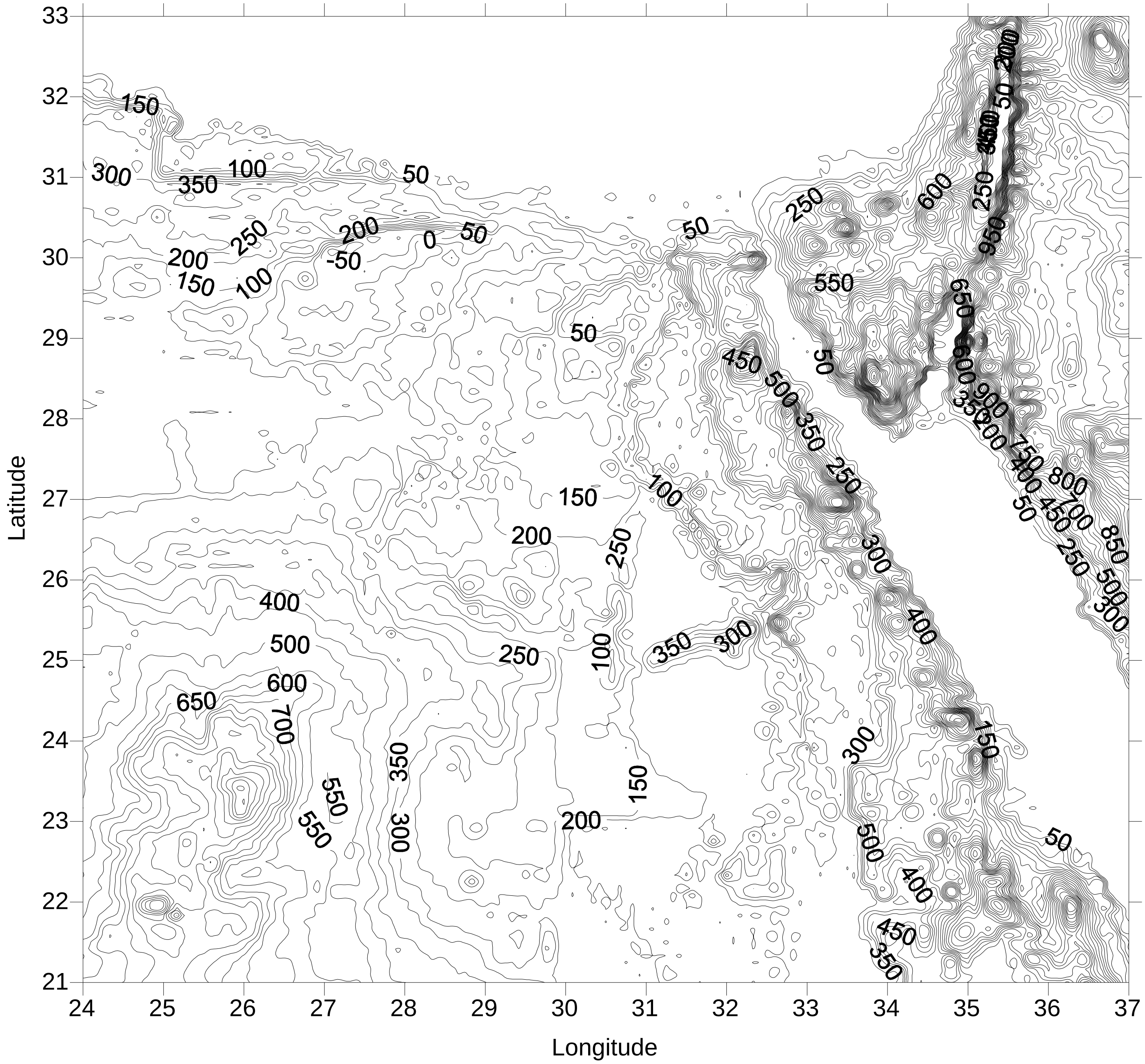

Figure (5b): Final topographic contour map for Egypt
(sea bathymetry neglected) (interval: 50m)

Figure (5)

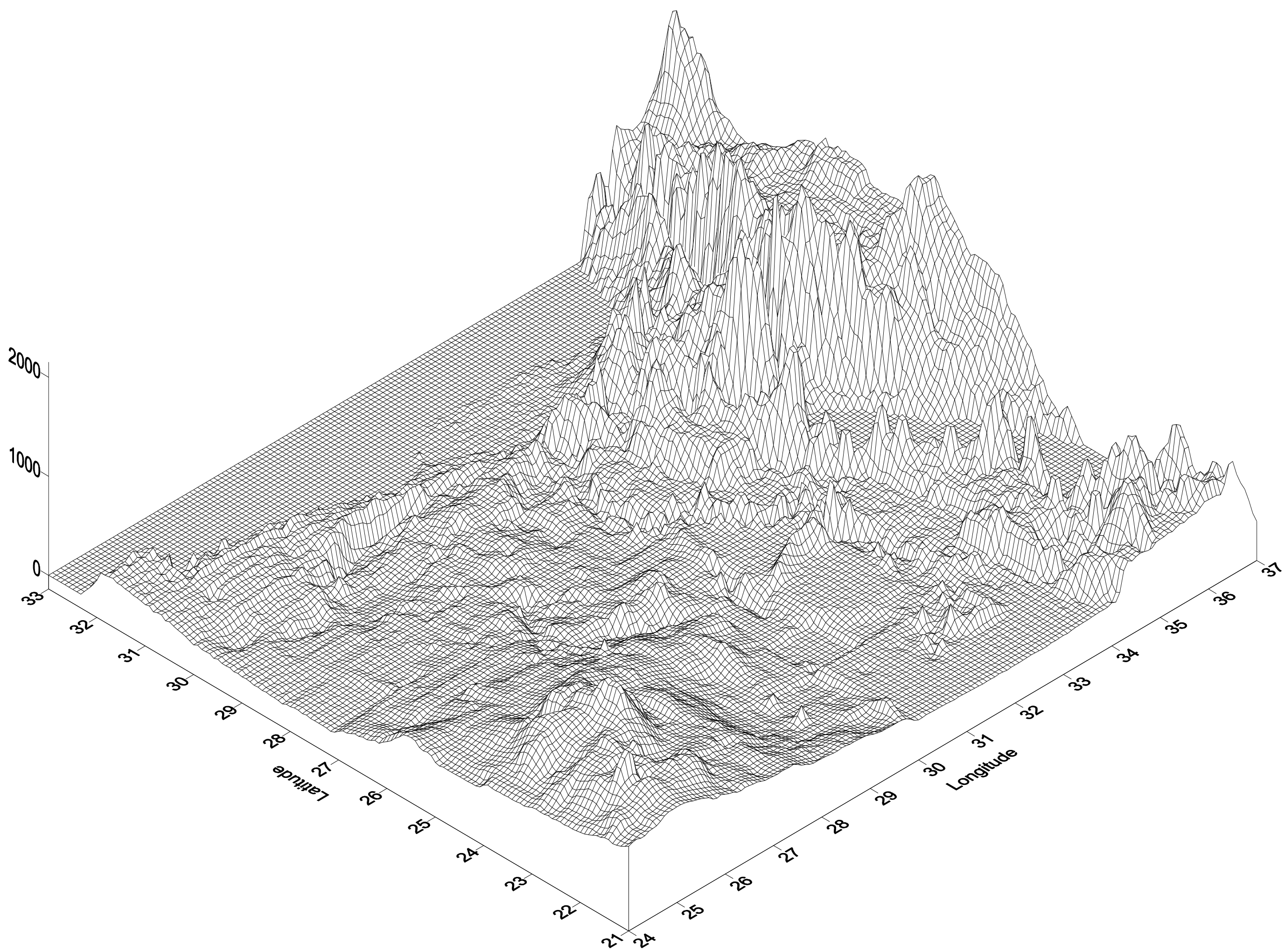

Figure (6): Topographic surface map for Egypt

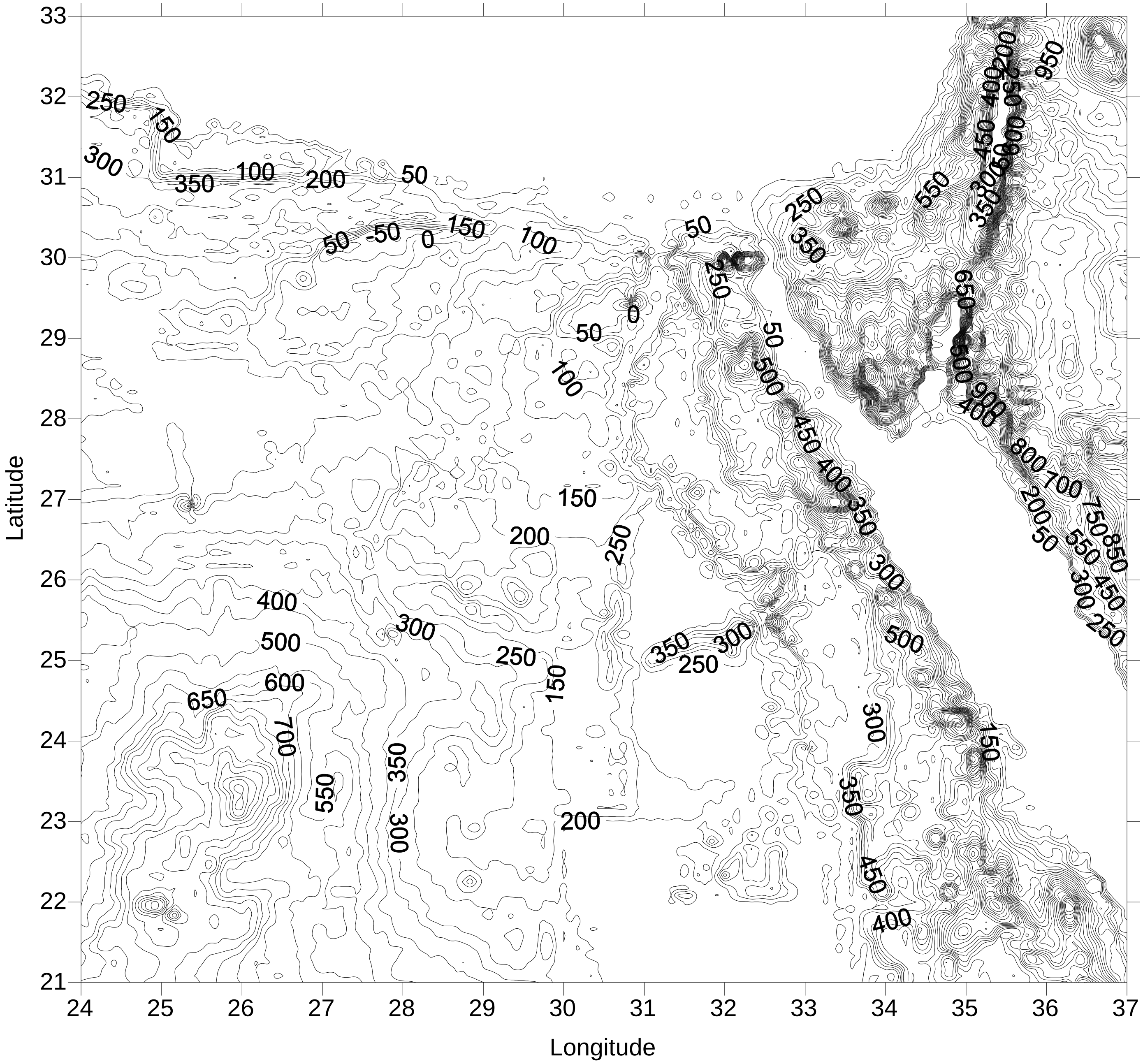

Figure (7): Final topographic contour map for Egypt corresponding to the addition of new data points (sea bathymetry neglected) (interval: 50m)

References

Abd-Elmotaal, H. and El-Tokhey, M. (1997): "Detection of gross errors in the gravity net in Egypt", Survey Review, 34, 266: 223-228.

Burša, M. (1971): "Single-Layer Density as Function of Stokes' Constants", Studia geoph. et geod., 15: 113-123..

Fan, H. (1998): "On an Earth ellipsoid best-fitted to the Earth surface", Journal of Geodesy, 72: 511-515.

Heiskanen, W.A. and Moritz, H. (1967): "Physical Geodesy", W.H. Freeman and Company, San Francisco and London.

Moritz, H. (1980): "Advanced Physical Geodesy", Herbert Wichmann Verlag, Karlsruhe.

Tscherning, C.C. (1982): "Determination of a (quasi) geoid for the Nordic Countries from heterogeneous data using collocation", Paper presented at the meeting of the Nordic Geodetic Commission, Gävle, Sweden, September.

Tscherning, C.C. and Forsberg, R. (1986): "Geoid determination in the Nordic Countries from gravity and height data", Paper presented at the International Symposium on the Definition of the Geoid, Firenze, May 26-30.

Tscherning, C.C. (1999): "Construction of an-isotropic covariance functions using Riesz-representers", Journal of Geodesy, 73: 333-336.

Wenzel, G. (1998): "Ultra High Degree Geopotential Model GPM3E97A to Degree and Order 1800 tailored to Europe", Paper presented in Proceedings of 2nd Continental Workshop on the Geoid in Europe, March 10-14, Budapest.